Chapter End Test

Ch (1) & (2)

1. $f(x) = (2x + 3)^2$ for x > 0
   a. Find the range of f. [1]
   b. Find $f^{-1}(x)$. [3]
   c. State the domain of f^{-1}. [1]
   d. Find $ff(1)$. [2]

(a) $f(x) = (2x+3)^2$

$= (0+3)^2 = 9$

$y > 9$

(b) $y = (2x+3)^2$

$x = (2y+3)^2$

$\sqrt{x} = 2y + 3$

$\frac{\sqrt{x} - 3}{2} = y$

$f^{-1}(x) = \frac{\sqrt{x} - 3}{2}$,

(c) $x > 9$

(d) $ff(1) = f(2+3)^2$

$= f(25)$

$= (25 \times 2 + 3)^2$

$= (53)^2$

$= 2809$

2. (a) Write $2x^2 + 3x - 4$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

$2x^2 + 3x - 4 = a(x^2 + 2bx + b^2) + c$

$= ax^2 + 2abx + ab^2 + c$

$a = 2$

$2ab = 3$

$4b = 3$

$b = \frac{3}{4}$

$ab^2 + c = -4$

$2 \times \frac{9}{16} + c = -4$

$\frac{9}{8} + c = -4$

$c = -4 - \frac{9}{8}$

$= -4\frac{9}{8}$

$= -5\frac{1}{8}$

$\therefore 2x^2 + 3x - 4 = 2(x + \frac{3}{4})^2 - \frac{41}{8}$

(b) Hence, write down the coordinates of the stationary point on the curve $y = 2x^2 + 3x - 4$. [2]

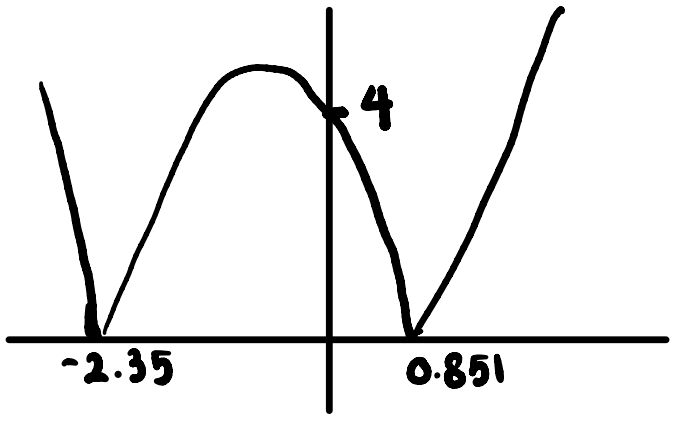
$(-\frac{3}{4}, -\frac{41}{8})$

(c) Sketch the graph of $y = |2x^2 + 3x - 4|$, showing the exact values of the intercepts of the curve with the coordinate axes. [3]

$x = 0, y = -4$

$y = 0, 0 = 2x^2 + 3x - 4$

$x = 0.851, x = -2.35$



(d) Find the value of k for which $\left|2x^2 + 3x - 4\right| = k$ has exactly 3 values of x. [1]

$$k = \frac{41}{8}$$

3. Find the value of k for which the line $y = kx - 7$ and the curve $y = 3x^2 + 8x + 5$ do not intersect. [6]

$b^2 - 4ac < 0$

$kx - 7 = 3x^2 + 8x + 5$

$0 = 3x^2 + 8x - kx + 5 + 7$

$a = 3, b = 8 - k, c = 12$

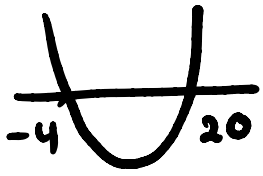
$b^2 - 4ac < 0$

$(8-k)^2 - 4(3)(12) < 0$

$64 - 16k + k^2 - 144 < 0$

$k^2 - 16k - 80 < 0$

$(k - 20)(k + 4) < 0$



$-4 < k < 20$

4. Find the set of values of k for which $4x^2 - 4kx + 2k + 3 = 0$ has no real roots.
[5]

$b^2-4ac<0$

$a=4,\ b=-4k,\ c=2k+3$

$b^2-4ac<0$

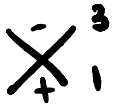
$(-4k)^2-4(4)(2k+3)<0$

$16k^2-16(2k+3)<0$

$16k^2-32k-48<0$

$(\div 16)$

$k^2-2k-3<0$



$(k-3)(k+1)<0$

$-1<k<3$

5. Solve the equations

$$y - x = 4, \rightarrow y = 4 + x$$

$$x^2 + y^2 - 8x - 4y - 16 = 0.$$

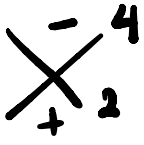
[5]

$x^2+(4+x)^2-8x-4(4+x)-16=0$

$x^2+16+8x+x^2-8x-16-4x-16=0$

$2x^2-4x-16=0$

$\div 2$ $x^2-2x-8=0$



$(x-4)(x+2)=0$

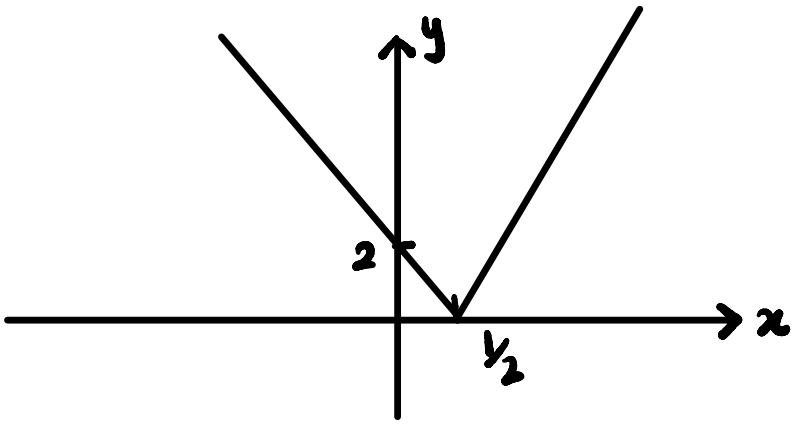
$x=4$ or $x=-2$

$y=8$ $y=2$

6. (i) Sketch the graph of $y = |4x - 2|$ on the axes, showing the coordinates of the points where the graph meets the axes. [3]

$x = 0,\ y = -2$

$y = 0,\ x = \frac{1}{2}$



(ii) Solve the equation $|4x - 2| = x$. [3]

$4x - 2 = x$ or $4x - 2 = -x$

$3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

$5x - 2 = 0$

$5x = 2$

$x = \frac{2}{5}$